Version of June 17, 2007

# A list of corrections and suggested additions to my book "Functional Analysis for Probability and Stochastic Processes"

*Many thanks to Dirk Werner, Radek Bogucki, Wilbur Langson and many others for supplying items to this list.*

- Page 8 lines 7 and 8 from the top: change "$\limsup$" to "$\liminf$" (twice).
- Page 32: Add the following exercise after 1.5.8: Let $(\Omega, \mathcal{F}, \mu)$ be a measure space, and let $x_i, i = 1, ..., n$ be measurable functions on $\Omega$ with values in $[0, \infty]$. Then $\int_\Omega x_1^{k_i} ... x_n^{k_n} \,\mathrm{d}\mu \le \left(\int_\Omega x_1^k \,\mathrm{d}\mu\right)^{\frac{k_1}{k}} ... \left(\int_\Omega x_n^k \,\mathrm{d}\mu\right)^{\frac{k_n}{k}}$, for positive $k_i, i = 1, 2, ..., n$ and $k = \sum_{i=1}^n k_i$.
- Page 39, line 3 from the top: Change "$\mathcal{D}(A)$" to "$\mathcal{D}(L)$"..
- Page 65, line 14 from the top: Change "$A_\lambda$ belongs" to "$A_\lambda x$ belongs".
- Page 65, line 1 from the bottom: Change "since" to "for".
- Page 66: Add an exercise saying: Let $\mathbb{X} = \mathbb{R}^2$ with $\|(\xi_1, \xi_2)\| = |\xi_1| + |\xi_2|$. Check that for any operator $A$ in $\mathbb{X}$ there exists a $2 \times 2$ matrix $M = (\alpha_{ij})_{i,j=1,2}$ such that $A(\xi_1, \xi_2) = (\xi_1, \xi_2)M$ (matrix multiplication). Also, in such a case: $\|A\| = (|\alpha_{11}| + |\alpha_{12}|) \vee (|\alpha_{21}| + |\alpha_{22}|)$.
- Page 81, subsection 3.1.3, Change "Schwartz" to "Schwarz". The same applies to Exercise 3.3.7 on page 95, to line 17 from the top of page 143, and to line 10 from the bottom of page 391.
- Page 83, line 14 from the top: Change "Let $z$ belong" to "Let $z \neq 0$ belong".
- Page 83, line 15 from the top: Change "$+t^2\|z\|$" to "$+t^2\|z\|^2$".
- Page 83, line 16 from the top: Change "$-\frac{2(z,x-y)}{\|x-y\|^2}$" to "$-\frac{(z,x-y)}{\|z\|^2}$".
- Page 97, line 9 from the bottom: Change "minimize the distance $d(Z, X)$" to "minimize the distance $d(Z, g(X))$".
- Change Exercise 3.3.16 to read: "Let $N_1$ and $N_2$ be two independent, Poisson distributed random variables with parameters $\lambda$ and $\mu$, respectively. Show that $\mathbb{E}\,(N_1|N_1 + N_2) = \frac{\lambda}{\lambda+\mu}(N_1 + N_2)$. More generally, conditional distribution of $N_1$ given $N_1 + N_2$ is binomial with parameters $n = N_1 + N_2$ and $p = \frac{\lambda}{\lambda+\mu}$ (see 3.3.17)."
- Page 106, line 6 from the top: Change "$p$" to "$\frac{k}{2N}$" (twice).
- Page 106, line 6 from the top: Change "$X(t) = k$" to "$X(n) = k$".
- Page 136, lines 11 and 14 from the top: Change "$\mathbb{P}\{\lim_{t\to 0} tw_{\frac{1}{t}} = 0\}$" to "$\mathbb{P}\{\lim_{t\to 0} tw_{\frac{1}{t}} = 0\} = 1$".
- Page 154, line 15 from the bottom: Change "$l := \limsup$" to "$l := \liminf$".
- Page 154, lines 13-14: Change "$\limsup_{n\to\infty}(-\xi_n) = -\liminf_{n\to\infty} \xi_n$" to $\liminf_{n\to\infty} (-\xi_n) = -\limsup_{n\to\infty} \xi_n$".
- Page 162, line 6 from the bottom: Change "$L^p(\Omega, \mathcal{F}, \mu), p > 1$, is isomorphic to" to "$[L^p(\Omega, \mathcal{F}, \mu)]^*, p > 1$, is isometrically isomorphic to".
- Page 163, line 7 from the bottom should read: "$= \int \tilde{T}_\mu x \,\mathrm{d}\nu = < \nu, \tilde{T}_\mu x >$".

- Page 163, line 6 from the bottom should read: "which proves that $S_\mu$ is dual to $\tilde{T}_\mu$. Analogously, $\tilde{S}_\mu$ is dual to $T_\mu$. The".
- Page 168, line 1 from the top: Change "the only known" to "the only known (classical)".
- Page 168, Exercise 5.4.3: This should read "Consider the same sequence $e_k, k \geq 1$ in the Hilbert space $l^2$ of square integrable sequences, and prove that it converges weakly but not strongly to 0."
- Page 186, lines 2 and 3 from the bottom: Change "Let $S$ be a compact topological space," to "Let $S$ be a compact topological space such that $\mathbb{PM}(S)$ with weak$^*$ topology can be made into a metric space (as in 5.6.5, for example),".
- Page 193, lines 7 and 6 from the bottom: Change "To show sufficiency, for $i \in \mathbb{N}$," to "To show sufficiency, by Lemma 5.7.18, it is enough to assume that $A$ is closed and prove that it is compact. To this end, for $i \in \mathbb{N}$,".
- Page 193, line 3 from the top: Change "If a $x_n \in C(\mathbb{R}^+), n \geq 1$ does not" to "If a sequence $x_n, n \geq 1$ of elements of $C(\mathbb{R}^+)$ does not".
- Page 194, line 10 from the top: Change

$$\text{“}\sup_s \mathbb{P}\left\{\sup_{s\leq u\leq s+h} |\mathcal{X}_n(u) - \mathcal{X}_n(s)|\right\}\text{”}$$

  to

$$\text{“}\sup_s \mathbb{P}\left\{\sup_{s\leq u\leq s+h} |\mathcal{X}_n(u) - \mathcal{X}_n(s)| > \epsilon\right\}\text{”}.$$

- Page 194, line 14 from the top: Change "equals $\mathbb{P}\{S_m > \sqrt{n}\epsilon\}$" to "equals $\mathbb{P}\{S_m^* > \sqrt{n}\epsilon\}$".
- Page 194, line 17 from the top: Change

$$\text{“}\mathbb{P}(S_m^* \geq 2\sqrt{m}r) \leq \frac{2\mathbb{P}\{S_m \geq \sqrt{m}r\}}{1 - r^{-2}}\text{”}$$

  to

$$\text{“}\mathbb{P}(S_m^* \geq 2\sqrt{m}r) \leq \frac{\mathbb{P}\{|S_m| \geq \sqrt{m}r\}}{1 - r^{-2}}\text{”}.$$

- Page 195, line 9 from the top: Change

$$\text{“}\limsup_{n\to\infty} \sup_{s\geq 0} \mathbb{P}\left\{\sup_{s\leq u\leq s+h} |\mathcal{X}_n(u) - \mathcal{X}_n(s)|\right\}\text{”}$$

  to

$$\text{“}\limsup_{n\to\infty} \sup_{s\geq 0} \mathbb{P}\left\{\sup_{s\leq u\leq s+h} |\mathcal{X}_n(u) - \mathcal{X}_n(s)| > \epsilon\right\}\text{”}.$$

- Page 196, line 3 from the bottom: Change "$\frac{S_{[nt_k]} - S_{[nt_{k-1}]}}{\sqrt{t_2 - t_1}}$" to "$\frac{S_{[nt_k]} - S_{[nt_{k-1}]}}{\sqrt{t_k - t_{k-1}}}$".
- Page 198, line 20 from the bottom: Add "for any $\epsilon > 0$,".
- Page 203, line 8 from the bottom: Change "$L^($$\mathbb{R}^+)$" to "$L^1(\mathbb{R}^+)$".
- Page 211, line 12 from the top: Change "$\alpha(\sigma) = \alpha(\sigma)^2 \geq 0$" to "$\alpha(\sigma) = \alpha(\frac{\sigma}{2})^2 \geq 0$".
- Page 232, line 2 from the top: Change "$i_\lambda \phi$" to "$i_\lambda * \phi$".
- Page 249, line 11 from the top: Change "$T_t = e^{\omega t} x$" to "$T_t x = e^{\omega t} x$".
- Page 250, line 11 from the bottom: Change "$x(\tau + t) - x(t)$" to "$x(\tau + h) - x(\tau)$".

- Page 250, line 10 from the bottom: Change "exists (even uniformly in $\tau \geq 0$). Therefore, $x \in \mathcal{D}(A)$ must be" to "exists (even uniformly in $\tau \geq 0$), proving that $x$ is differentiable from the right with continuous derivative. This implies that $x \in \mathcal{D}(A)$ must be".
- Page 253: Add the following exercise after 7.4.21: For $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ check that $\mathrm{e}^{tA} = \mathrm{e}^{rt}\left[\frac{\sinh(pt)}{p} + (\cosh(pt) - \frac{r}{p}\sinh(pt))I\right]$ where $p = \sqrt{r^2 - d}, r = \frac{a+d}{2}, d = \det A = ad - bc$, provided $p \neq 0$. For $p = 0$ we have $\mathrm{e}^{rt}(tA + (1 - rt)I)$, where $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. Hint: use the Cayley-Hamilton Theorem.
- Page 254: Add the following exercise after 7.4.24: (a) Let $\mathbb{X} = C[0,1]$. Show that the semigroups $\{S_t^i, t \geq 0\}, i = 0,1$ given by $S_t^i x(\tau) = x(\tau \mathrm{e}^{-t} + i(1 - \mathrm{e}^{-t}))$ are isomorphic. (b) Show that $\{S_t^0, t \geq 0\}$ is isomorphic to the semigroup of left translations in $C[0,\infty]$. (c) Use (a) and (b) to characterize the infinitesimal generators of $\{S_t^i, t \geq 0\}, i = 0,1$.
- Page 255: Add the following exercise after 7.4.27: Find the general form of the exponent of a $2 \times 2$ intensity matrix. Hint: diagonalize $\begin{pmatrix} -\alpha & \alpha \\ \beta & -\beta \end{pmatrix}$ to obtain that its exponent equals $\begin{pmatrix} \beta + \alpha\mathrm{e}^{-t} & \alpha - \alpha\mathrm{e}^{-t} \\ \beta - \beta\mathrm{e}^{-t} & \beta\mathrm{e}^{-t} + \alpha \end{pmatrix}, t \in \mathbb{R}$. Alternatively, use the (new) exercise from page 253.
- Page 268, formula (7.42): change "$\lambda$" to "$a$" (twice).
- Page 291, line 5 from the bottom: Change "$(\alpha_1, \alpha_2, \alpha_3, \alpha_4)^{\mathrm{T}}$" to "$\frac{1}{4}(\alpha_1, \alpha_2, \alpha_3, \alpha_4)^{\mathrm{T}}$".
- Page 299, line 9 from the top: Change "$K(t, \tau, B)$" to "$K(t, p, B)$".
- Page 299, line 4 from the bottom: Change "$p_{n,m} \geq 0$ and $\sum_{m \geq 1} p_{n,m} = 1$" to "$p_{n,m}(t) \geq 0$ and $\sum_{k \geq 1} p_{n,k}(t) = 1$ for all $n, m \geq 1, t \geq 0$".
- Page 299, lines 2 and 3 from the bottom: Change "$p_{n,m}(s+t) = (...) = \sum_{m \geq 1} p_{m,n}(s) p_{n,m}(t)$" to "$p_{n,m}(s+t) = K(s+t, n, \{m\}) = \int_{\mathbb{N}} K(s, i, \{m\})\, K(t, n, \mathrm{d}i) = \sum_{i \geq 1} p_{i,m}(s) p_{n,i}(t)$".
- Page 302, line 6 from the top: Change "$(x, U_t) = (T_t x, \mu)$" to "$(x, U_t \mu) = (T_t x, \mu)$".
- Page 302, line 10 from the bottom: Change "$l^1$" to "$l^\infty$".
- Page 306, line 11 from the top: Delete "(sometimes called a non-accessible boundary)".
- Page 310: Replace the full stop with "for $\lambda > 0$." in formula (8.30).
- Page 311: Change "$x \in \mathcal{D}(A)$" to "$x \in \mathcal{D}(A_p)$" in formula (8.34).
- Page 311, line 2 from the top: Change

$$\text{“}\|\mathrm{e}^{\lambda^2 R_\lambda t}\| \leq \sum_{n=0}^{\infty} \frac{\|\lambda^{2n} R_\lambda^n\|}{n!} \leq M \sum_{n=0}^{\infty} \frac{\lambda^n}{n!} = \mathrm{e}^{\lambda t} M.\text{”}$$

to

$$\text{“}\|\mathrm{e}^{\lambda^2 R_\lambda t}\| \leq \sum_{n=0}^{\infty} \frac{\|\lambda^{2n} t^n R_\lambda^n\|}{n!} \leq M \sum_{n=0}^{\infty} \frac{\lambda^n t^n}{n!} = \mathrm{e}^{\lambda t} M.\text{”}$$

- Page 318, line 15 from the bottom: Change "$U(t)\lambda R_\lambda$" to "$U(t)\lambda R_\lambda x$".
- Page 334, line 6 from the top: Change "the range of $A$ is dense" to "the range of $\lambda - A$ is dense".

- Page 334, line 25 from the top: Instead of reference to [65] there should be the reference to the original Kato's paper "On the semi-groups generated by Kolmogoroff's differential equations", J. Math. Soc. Japan, 6 (1954), 1-15.
- Page 345, line 18 from the top: Change "$\mathbb{X}$" to "$\mathbb{X}'$".
- Page 345: Add the following exercise after 8.4.7: Let $B_i, i = 1, 2$ be two closed operators in a Banach space $\mathbb{X}$ and let $C_i, i = 1, 2$ be two bounded operators in this space such that $C_1 + C_2 = I_{\mathbb{X}}$. Consider the operators $A_n$ in $\mathbb{X}\times\mathbb{X}$ given by $A_n\binom{x}{y} = \binom{B_1x+nC_1(y-x)}{B_2y+nC_2(x-y)}, n \geq 1$ defined on $\mathcal{D}(B_1)\times\mathcal{D}(B_2)$. Show that $\mathcal{D}(A_{ex}) \subset \{\binom{x}{y} \in \mathbb{X} \times \mathbb{X} | x = y\}$.
- Page 349, lines 3 and 6 from the bottom: Change "$x \in \mathbb{X}$" to "$x \in \mathbb{X}'$".
- Page 350, line 11 from the bottom: Change "by 8.4.9" to "by 8.4.9, 8.4.10 and 8.4.17".
- Page 357, line 10 from the top: Change "by 8.4.9" to "by 8.4.9, 8.4.10 and 8.4.17".
- Page 358, line 7 from the top: Change "distribution of $X_{t/n}, X_{2t/n}-X_{t/n}, ...,$ $X_t-X_{(n-1)t/n}$," to "distribution of $X_t-X_{(n-1)t/n}, \ldots, X_{2t/n}-X_{t/n}, X_{t/n}$,".
- Page 374, change lines 6 and 7 from the bottom to read: "*Hint to Exercise 3.3.16* $\mathbb{P}(N_1 = k|N_1 + N_2 = l) = \frac{\mathbb{P}(N_1=k)\mathbb{P}(N_2=l-k)}{\mathbb{P}(N_1+N_2=l)}$."
- Page 376, Hint to Exercise 5.4.3 should read: "To prove weak convergence use 3.1.28."